37242 Introduction to Optimisation

Tutorial 9

1. (slightly modified from Nash and Sofer, p.434)

   Consider the problem

$$\begin{array}{ll} \min & f(\mathbf{x}) = x_1^2 + x_1^2 x_3^2 + 2x_1 x_2 + x_2^4 + 8x_2 \\ \text{s.t.} & 2x_1 + 5x_2 + x_3 = 3 \end{array}$$

   (a) Determine which of the following points are stationary points:

$$\text{(i)} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{(ii)} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \text{ and (iii)} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

   (b) Determine whether each stationary point is a local minimiser.

2. Consider the problem

$$\begin{array}{lrcrcrcl} \min & x_1^2 + 3x_1x_2 + 9x_2^2 + x_3^2 & & & & & & \\ \text{s.t.} & x_1 & - & x_2 & + & x_3 & = & 4 \\ & 2x_1 & + & x_2 & + & 5x_3 & = & 8 \end{array}$$

   (a) Write down the Lagrangian function for this problem.

   (b) Use the Lagrangian to confirm that the solution $(x_1^*, x_2^*, x_3^*) = (2.6, -0.7, 0.7)$ is a stationary point for the above constrained optimisation problem (you will need to find optimal values for the Lagrange multipliers μ_1 and μ_2).

3. Consider the problem

$$\begin{array}{ll} \min & f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2 + 4x_1. \\ \text{s.t.} & x_1 + 3x_2 = 1 \end{array}$$

   (a) Write down the Lagrangian function for this problem.

   (b) Use the Lagrangian to find all stationary points for this problem.

4. (slightly modified from Nash and Sofer, p.437)

Consider the problem

$$\begin{array}{ll} \min & f(x_1, x_2, x_3) = 3x_1^2 - \frac{1}{2}x_2^2 - \frac{1}{2}x_3^2 + x_1x_2 - x_1x_3 + 2x_2x_3 \\ \text{s.t.} & 2x_1 - x_2 + x_3 = 2. \end{array}$$

(a) Find a stationary point for this problem using the Lagrangian.

(b) Show that this is a local minimum.