37242 Introduction to Optimisation

Tutorial 8

Question 1. Solve the unconstrained optimisation problem

$$\min f(x_1,\, x_2) = x_1^2 + x_2^2 + x_1x_2 + 4x_1$$

(a) By steepest descent method
(b) By Newton's method
(c)By finding stationary points and determining their nature

Extra excercises:

Question 2. Solve the unconstrained optimisation problem with the methods above

$$\min f(x_1,\, x_2) = x_1^2 + x_2^2 + 4x_1 - 6x_2$$

Question 3. Prove that if A is an n × n positive definite matrix, then

(a) All eigenvalues of $\mathbf{A}$ are positive.

(b) $\mathbf{A}$ is invertible.

(c) All eigenvalues of $\mathbf{A}^{-1}$ are positive.

Question 4. (Winston Chapter 11, Section 3, Question 1, 2, 7, 8, 9)

On the given set $\mathbf{S}$, determine whether each function is convex, concave, or neither.

(a) $f(x) = x^3$; $\mathbf{S} = [0,\, \infty)$.

(b) $f(x) = x^3$; $\mathbf{S} = \mathbf{R}$.

(c) $f(x_1,\, x_2) = x_1^2 + x_2^2$; $\mathbf{S} = \mathbf{R}^2$.

(d) $f(x_1,\, x_2) = -x_1^2 - x_1x_2 - 2x_2^2$; $\mathbf{S} = \mathbf{R}^2$.

(e) $f(x_1,\, x_2,\, x_3) = -x_1^2 - x_2^2 - 2x_3^2 + 0.5x_1x_2$; $\mathbf{S} = \mathbf{R}^3$.