

## Class Test 1

St.Id:

Name:

There are three questions in this test, total time 45 min.

Q1. Consider the following LP problem:

(6 marks)

$$\max z = 5x_1 + x_2$$

$$\text{s.t. } x_1 - x_2 \leq 5$$

$$0.6x_1 + x_2 \leq 15$$

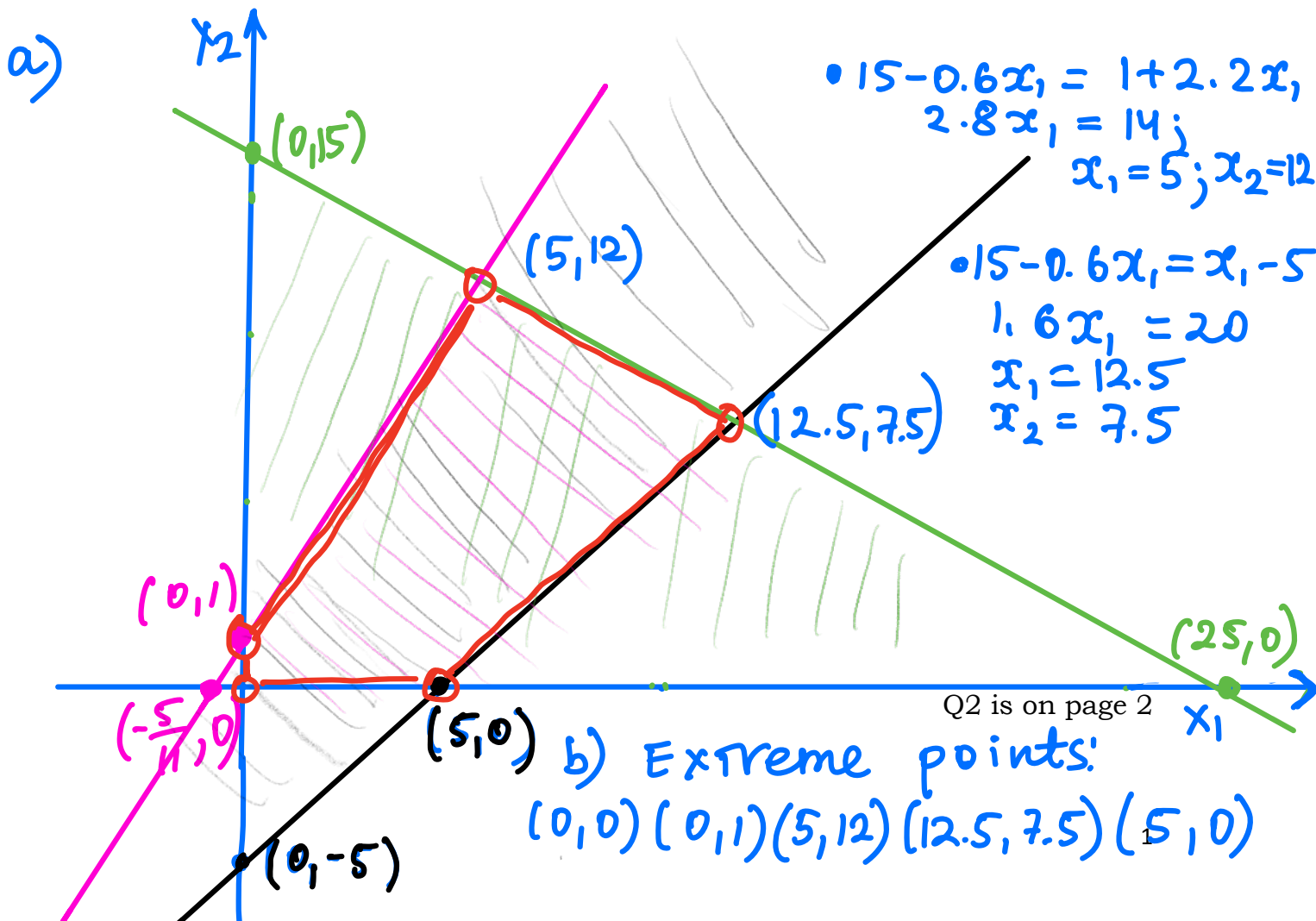
$$-2.2x_1 + x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

$$\begin{aligned} \rightarrow & \bullet x_2 \geq x_1 - 5 \\ \rightarrow & \bullet x_2 \leq 15 - 0.6x_1 \\ & \bullet x_2 \leq 1 + 2.2x_1 \end{aligned}$$

- Graph the feasible region of this LP
- Find its extreme points – please specify their coordinates
- What is the optimal value of the objective function and what is the optimal solution?

Please provide your solution below:



$$c) \quad z = 5x_1 + x_2$$

$$z(0,0) = 0 \quad z(12.5, 7.5) = 70$$

$$z(0,1) = 1 \quad z(5,0) = 25$$

$$z(5,12) = 37$$

$$\text{max } z = 70 ; \quad x = \begin{pmatrix} 12.5 \\ 7.5 \end{pmatrix}$$

Q2. Consider the following LP problem:

(6 marks)

$$\begin{aligned} \min z &= -3x_1 + 8x_2 \\ \text{s.t.} \quad &4x_1 + 2x_2 \leq 10 \\ &2x_1 + 3x_2 \leq 6 \\ &x_1, x_2 \geq 0 \end{aligned}$$

- Write the problem in a standard form
- Specify the initial basis
- Solve the problem by the Simplex method in tabular form, show the detailed working.
- Specify the optimal value of objective function and the optimal solution.  
Please provide your solution below:

a)  $\min z = -3x_1 + 8x_2 \rightarrow z + 3x_1 - 8x_2 = 0.$   
 s.t.  $4x_1 + 2x_2 + s_1 = 10$   
 $2x_1 + 3x_2 + s_2 = 6.$   
 $x_1, x_2, s_1, s_2 \geq 0.$

b)  $x_{B_0} = (s_1, s_2)$

	$x_1$	$x_2$	$s_1$	$s_2$	RHS
$z$	3	-8	0	0	0
$s_1$	4	2	1	0	10
$s_2$	2	3	0	1	6
$z$	0	<del><math>-\frac{19}{2}</math></del>	<del><math>-\frac{3}{4}</math></del>	0	<del><math>-\frac{15}{2}</math></del>
$x_1$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{5}{2}$
$s_2$	0	2	$-\frac{1}{2}$	1	1

Ratio  
 $10/4 = \frac{5}{2} < 3$   
 $6/2 = 3$

$R_0' = R_0 - 3R_1'$   
 $R_1' = R_1/4$

$R_2' = R_2 - 2R_1'$

Q3 is on page 3

Optimal, as all  $\hat{C}_N < 0$   
 $z^* = -\frac{15}{2}; x^* = \begin{pmatrix} 5/2 \\ 0 \end{pmatrix}$

Q3. Consider the set of vectors  $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$  satisfying (3 marks)

$$3x_1 + 5x_2 \leq 3, \quad x_1, x_2 \geq 0$$

- a) Write the inequality above in the form  $ax \leq b$ , where  $a$  is a vector and  $b$  is a constant  
 b) Prove that this set is convex.

Please provide your solution below:

$$a) \quad (3, 5) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq 3$$

$$b) \quad \text{Let } x' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}, \quad x'' = \begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix} :$$

$$(3, 5)x' \leq 3 \quad (3, 5)x'' \leq 3$$

$$\tilde{x} = \alpha x' + (1-\alpha)x''$$

$$(3, 5)\tilde{x} = \alpha(3, 5)x' + (1-\alpha)(3, 5)x'' \leq 0$$

$$\alpha \times 3 + (1-\alpha) \times 3 = 3$$

$$\downarrow$$

$$(3, 5)\tilde{x} \leq 3 \rightarrow \text{the set is convex}$$

### Table of formulae for Class Test 1

- Convex set: A set  $S$  in  $n$ -dimensional space is *convex* if for any two points  $x^{(1)}$  and  $x^{(2)}$  from  $S$ , any point of the line segment connecting  $x^{(1)}$  and  $x^{(2)}$  also belongs to  $S$ . In other words, a set  $S$  is a convex set if the line segment joining any pair of points in  $S$  is wholly contained in  $S$ : for any two points  $x^{(1)}$  and  $x^{(2)}$  in  $S$  and  $\alpha \in (0,1)$ ,  $x^* = \alpha x^{(1)} + (1 - \alpha) x^{(2)}$ :  $x^* \in S$

#### Full Simplex tableau:

basis	$x_N$	$x_B$	rhs
$z$	$c_B^T B^{-1} N - c_N^T$	$0^T$	$c_B^T B^{-1} b$
$x_B$	$B^{-1} N$	<b>I</b>	$B^{-1} b$