University of Technology Sydney
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Mathematical Statistics (37262) – Class 5 Preparation Work SOLUTIONS

1. i) One possible substitution is $y = 1 - e^{2-x}$ and hence

$2 - \ln(1-y) = x$ and $\dfrac{dy}{dx} = e^{2-x} = 1-y$.

Given N independent realisations of $U \sim U[0,1]$ we could then estimate

$$\int_2^{\infty} \frac{3}{\ln(x)\left[\sin(x)+2\right]}dx \approx \frac{1}{N}\sum_{i=1}^{N} \frac{3}{(1-y)\ln(2-\ln(1-y))\left[\sin(2-\ln(1-y))+2\right]}$$

ii) Setting $y = \dfrac{x+0.1}{1.1}$ gives $\int_{-0.1}^{1} e^{x^4}dx = \int_0^1 1.1e^{(1.1y-0.1)^4}dy$.

Applying the Monte Carlo method as described above gives

$$\int_{-0.1}^{1} e^{x^4}dx \approx \frac{1}{5}\left[1.1e^{(1.1\times 0.654-0.1)^4} + \ldots + 1.1e^{(1.1\times 0.591-0.1)^4}\right] \approx 1.072.$$

iii) Setting $y = x+6$ gives $\int_{-6}^{-5} \arctan(x)dx = \int_0^1 \arctan(y-6)dy$

Applying the Monte Carlo method as described above gives

$$\int_{-6}^{-5} \arctan(x)dx \approx \frac{1}{5}\left[\arctan(0.654-6) + \ldots + \arctan(0.591-6)\right] \approx -1.393.$$

2.

i) For $y=\frac{1}{1+x^2}$, $x=-\sqrt{\frac{1}{y}-1}$ since x only takes non-positive values. We also have that $\frac{dx}{dy}=\frac{1}{2y^2\sqrt{\frac{1}{y}-1}}$ hence

$$\int_{-\infty}^{0}\left\lfloor\frac{10}{1+x^2}\right\rfloor dx=\int_{0}^{1}\lfloor 10y\rfloor\frac{1}{2y^2\sqrt{\frac{1}{y}-1}}dy$$

Applying the Monte Carlo method, we obtain

$$\int_{-\infty}^{0}\left\lfloor\frac{10}{1+x^2}\right\rfloor dx$$
$$\approx\frac{1}{5}\left[\lfloor 10\times 0.654\rfloor\frac{1}{2\times 0.654^2\sqrt{\frac{1}{0.654}-1}}+\ldots+\lfloor 10\times 0.591\rfloor\frac{1}{2\times 0.591^2\sqrt{\frac{1}{0.591}-1}}\right]$$
$$\approx 7.654$$

ii) We could also use, for example $y=e^x$, so $x=\ln(y)$ and $\frac{dy}{dx}=y$. This gives

$$\int_{-\infty}^{0}\left\lfloor\frac{10}{1+x^2}\right\rfloor dx=\int_{0}^{1}\left\lfloor\frac{10}{1+(\ln(y))^2}\right\rfloor\frac{1}{y}dy$$
$$\approx\frac{1}{5}\left[\left\lfloor\frac{10}{1+(\ln(0.654))^2}\right\rfloor\frac{1}{0.654}+\ldots+\left\lfloor\frac{10}{1+(\ln(0.591))^2}\right\rfloor\frac{1}{0.591}\right]\approx 9.654$$

3. Both the expectation and the variance of X are not finite, hence the Monte Carlo method cannot be used.

e.g. $E(X)=\int_{-\infty}^{-4}\frac{1}{x}dx+\int_{-4}^{4}\frac{x}{16}dx+\int_{4}^{\infty}\frac{1}{x}dx=\infty$.