

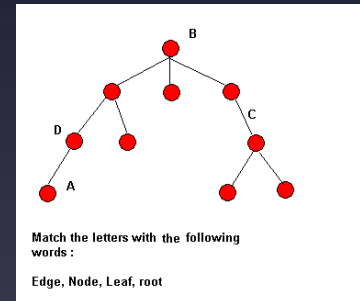
Week 07: Computer Science 1

Part 3: Number systems

• Recap: Theory!!

- This is a foundation subject

→ **You need to learn some computer science theory!!!**



- file systems

- complexity theory

- **information representation – binary, hex**

- Logic - Boolean algebra, binary arithmetic

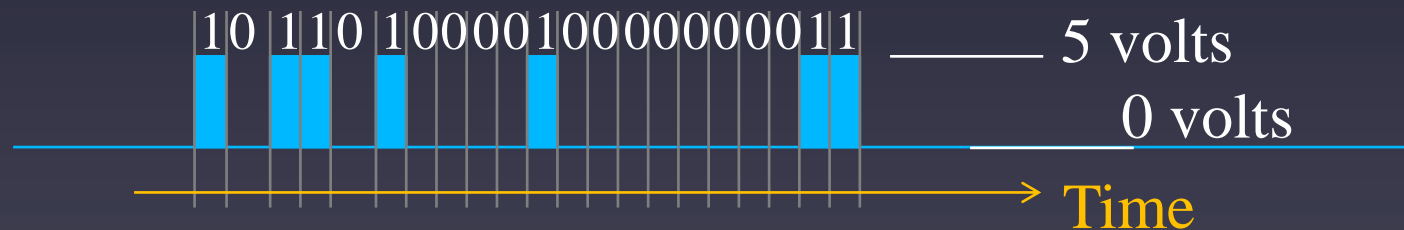
- computation theory, memory, coding

10101010 = 0xAA

$\text{not}(A \text{ and } B) = \text{not}(A) \text{ or } \text{not}(B)$

Numbers in conventional computers

- Data travels along an electrical conductor – typically wires, but now can be optical (or other media)
 - As 'pulse' of voltage – ie 0 volts or > 0 volts (eg 5V) at any given time
 - Assume 0 volts represents 0
 - Assume > 0 volts (eg 5V) represents 1
- A string of 1's and 0's is a binary number



- Note: Many circuits now use 3.6V, 1.8V or lower – needed because of increasing clock speed of chips & energy efficiency.

Numerical systems

- A numeral system is a way of representing information.
 - sometimes also called a number system
- Important numeral systems implemented on computers include binary, decimal, octal and hexadecimal.

Hindu-Arabic Numeral systems

- Most common numbering system:
 - Hindu-Arabic system – positional base
 - The base dictates digits available
- E.g.
 - Decimal is base 10
 - Binary is base 2
 - There are an infinite number of bases
 - In computing we use base 2,8,10,16

- The base indicates how many digits available.
- In base 10, we have digits 0-9
 - In base 8, we have digits 0-7
 - In base 2, we have digits 0-1
- Use a subscript to indicate base
- Eg: Octal: 1234_8
- Eg: Decimal: 314159_{10}
 - decimal is default ie. omit the 10

Base 10 - The Decimal System

- Uses ten symbols: **0,1,2,3,4,5,6,7,8,9**
- Each symbol represents a quantity:
 - 0** represents the absence of things
 - 1** represents a single thing
 - 2** represents a pair of things ...
- The decimal system is positional:
we can represent quantities greater than 9 by stringing symbols together:

Base 10 - The Decimal System

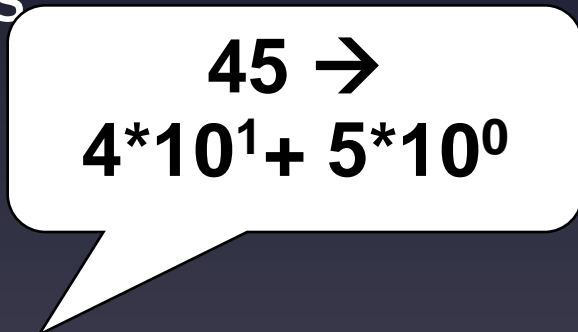
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$$45 \rightarrow 4 \cdot 10^1 + 5 \cdot 10^0$$

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234 →

$$2 \cdot 10^2 + 3 \cdot 10^1 + 4 \cdot 10^0$$

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Base 2 - The Binary System

- The binary system is positional like the decimal system
 - but it only has two symbols: **0** and **1**
- 10 represents $1*2^1 + 0*2^0$
(in decimal this is 2)
- 1101 represents $1*2^3 + 1*2^2 + 0*2^1 + 1*2^0$
(in decimal this is 13)

Joke:

- There are 10 types of people in this world.

Joke:

- There are 10 types of people in this world.
- *Those that can read binary and those that can't!*

Video

- <http://www.youtube.com/watch?v=fif6YVQakP4>

Discussion Questions

- What other number systems do we encounter in daily life?
- How do they differ from decimal and binary?

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- What other number systems do we encounter in daily life?
- How do they differ from decimal and binary?

How about Roman numerals?

How about Imperial measures? Inches?
Pounds? Clocks?

Using binary patterns: ASCII shortcuts

- Converting ASCII uppercase to lowercase
 - **A** is Decimal 65 which is 10000**01** in Binary
 - **a** is Decimal 97 which is 1**1**000**01** in Binary
 - **B** is Decimal 66 which is 10000**10** in Binary
 - **b** is Decimal 98 which is 1**1**000**10** in Binary
-
- Difference is always decimal 32 and means you change one bit in binary to change case.

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$$2^6 = 32$$


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Adding together binary numbers

1	0	1	1	0	1		-- A
0	1	0	1	0	1		-- B
1	1	1	1	0	1		-- Carry
<hr/>							
1	0	0	0	1	0		-- A + B

Exercises

Compute these binary sums:

- $011100 + 101011$

- $1011101 + 1000011$

- $100000 + 10110101$

- $1001101 + 1111$

Reflection on Symbols and Numbers

- Decimal: counting is 0,1,2,3,4,5,6,7,8,9...
 - the next number, 10 consists of two symbols - 1 and 0.
- Binary (or base 2) system only uses 2 symbols:
 - 0 and 1
- Hexadecimal (base 16) uses the symbols:
 - 0,1,2,3,4,5,6,7,8,9, A,B,C,D,E and F
 - A represents 10_{10} , B represents 11_{10} , F represents 15_{10}
- To represent any number in a particular base B, we need B symbols.

Reflection on Symbols and Numbers

- Exercises
 - How many symbols in a number system of base five? six? seven? eleven?

Reflection on Symbols and Numbers

- Exercises

- How many symbols in a number system of base five? six? seven? eleven?

- If a number system has the symbols

- ! @ # ^ \$ * ~ % { }

- What is its base?

- What decimal value would be !} ?

Binary-Decimal conversion

Binary 11100

$$= 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0$$

Binary-Decimal conversion

Binary 11100

$$\begin{aligned} &= 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0 \\ &= 16 + 8 + 4 + 0 + 0 \end{aligned}$$

Binary-Decimal conversion

Binary 11100

$$\begin{aligned} &= 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0 \\ &= 16 + 8 + 4 + 0 + 0 \\ &= 28 \end{aligned}$$

- Very easy!

Decimal-Binary conversion

Repeat: remove largest powers of 2

Example: Convert decimal 123 to binary

Decimal	Binary
$123 = 64 + 59$	$1000000 + \text{binary}(59)$

Summing the binary numbers: **1111011**

Decimal-Binary conversion

Repeat: remove largest powers of 2

Example: Convert decimal 123 to binary

Decimal	Binary
$123 = 64 + 59$	$1000000 + \text{binary}(59)$
$59 = 32 + 27$	$0100000 + \text{binary}(27)$
$27 = 16 + 11$	$0010000 + \text{binary}(11)$
$11 = 8 + 3$	$0001000 + \text{binary}(3)$
$3 = 2 + 1$	$0000010 + \text{binary}(1)$
$1 = 2^0$	0000001

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 **Summing the binary: 1111011**

Hexadecimal Numbers

- Numbers in base 16
- Sometimes referred to as “hex” numbers
- are often used as a shortcut for binary (you’ll see why shortly)

Hexadecimal Digits

Decimal	Binary	Hexadecimal
0	0	0
1	1	1
2	10	2
3	11	3
4	100	4
5	101	5
6	110	6
7	111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Another shortcut

- Octal is base 8
- Can be represented with 3 bits
- example of shortcut with chmod
- eg: `chmod 777 public_html`

- $7 = 111_2$
= rwx

Converting Hex to Decimal

- The conversion method is similar to that used to convert binary numbers to decimal
- We simply **multiply** each digit in our hex number by the appropriate power of **16**
- Let's convert hex number 7A25 to decimal....

Converting Hex to Decimal - 2

7	A	2	5				
			→	5	x	16 ⁰	= 5 x 1 = 5
		→	2	x	16 ¹	= 2 x 16 = 32	
	→	10	x	16 ²	= 10 x 256 = 2560		
→	7	x	16 ³	= 7 x 4096 = 28672			

Sum (Decimal) = 31269

Converting Decimal to Hexadecimal

- Similar to converting decimal to binary
- Three methods
 1. repeated division by 16 –or–
 2. repeated subtraction by 16 –or–
 3. convert to binary
then group the digits into four digits
then convert group to hex

Questions?

- ???

- You will convert number systems in the quiz and the final exam ...